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# GEOMETRY

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*Ann Xavier Gantert*



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## Dedication

This book is dedicated to Patricia Sullivan whose friendship and support has always been the mainstay of the author's life and work.

## Ann Xavier Gantert

The author has been associated with mathematics education in New York State as a teacher and an author throughout the many changes of the past fifty years. She has worked as a consultant to the Mathematics Bureau of the Department of Education in the development and writing of Sequential Mathematics and has been a coauthor of Amsco's *Integrated Mathematics* series, which accompanied that course of study.

## Reviewers:

Steven J. Balasiano  
*Assistant Principal,  
Supervision Mathematics  
Canarsie High School  
Brooklyn, NY*

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Eastchester, NY*

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Brooklyn, NY*

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Raymond Scacalossi Jr.  
*Mathematics Coordinator  
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Manhasset, NY*

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# PREFACE

**Geometry** is a new text for high school geometry that continues the approach that has made Amsco a leader in presenting mathematics in a contemporary, integrated manner. Over the past decades, this approach has undergone many changes and refinements to keep pace with the introduction and expansion of technology in the classroom.

Amsco texts parallel the integrated approach to the teaching of high school mathematics promoted by the National Council of Teachers of Mathematics in its *Curriculum and Evaluation Standards for School Mathematics*. In addition, the content of the book follows the guidelines mandated by the New York State Board of Regents in the *Mathematics Core Curriculum*. This book presents a range of materials and explanations to enable students to achieve a high level of excellence in their understanding of mathematics.

In this book:

- ✓ **Formal logic** is presented as the basis for geometric reasoning. Most of the geometric facts presented in this text are already familiar to the student. The purpose of this text is to help the student to use the principles of logic to understand the interdependence among these geometric and algebraic concepts.
- ✓ **Coordinate geometry** is presented with a postulational approach and used when appropriate to enhance and clarify synthetic proof.
- ✓ **Transformations** are introduced to further expand the students understanding of function and to relate that concept to geometry.
- ✓ **The concurrence theorems** for the altitudes, angle bisectors, medians and perpendicular bisectors of triangles are proved using a variety of approaches.
- ✓ **Solid geometry** is introduced and students are encouraged to expand their understanding of the three-dimensional world, particularly through the study of perpendicular and parallel lines and planes.
- ✓ **Algebraic skills** from *Integrated Algebra 1* are maintained, strengthened, and expanded as a bridge to *Algebra 2 and Trigonometry*.

- ✓ **Writing About Mathematics** encourages students to reflect on and justify mathematical conjectures, to discover counterexamples, and to express mathematical ideas in their own language.
- ✓ **Enrichment** is stressed both in the text and in the Teacher's Manual where many suggestions are given for teaching strategies and alternative assessment. The Manual provides opportunities for *Extended Tasks* and *Hands-On Activities*. Reproducible *Enrichment Activities* that challenge students to explore topics in greater depth are provided in each chapter of the Manual.

While *Integrated Algebra 1* is concerned with an intuitive approach to mathematics, the emphasis in *Geometry* is proof. In this text, geometry is developed as a postulational system of reasoning beginning with definitions, postulates, and the laws of reasoning. A unique blending occurs when students learn to apply the laws of logic to traditional deductive proof in geometry, both direct and indirect. The integration of traditional synthetic geometry, coordinate geometry, and transformational geometry is seen throughout the text and students learn to appreciate the interdependence of those branches of mathematics.

The intent of the author is to make the book of greatest service to the average student using thorough explanations and multiple examples. Each section provides careful step-by-step procedures for solving routine exercises as well as the non-routine applications of the material. Sufficient enrichment material is included to challenge students of all abilities. Specifically:

- ✓ Concepts are carefully developed using appropriate language and mathematical symbolism. General principles are stated clearly and concisely.
- ✓ Numerous examples are solved as models for students with detailed explanations of the mathematical concepts that underlie the solution. Where appropriate, alternative approaches are suggested.
- ✓ Varied and carefully graded exercises are given in abundance to develop skills and to encourage the application of those skills. Additional enrichment materials challenge the most capable students.

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