Relations
And
Functions

Day 1 Relations and Functions
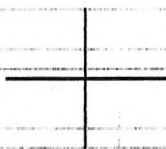
A relation is a set of ordered pairs.

What is an example of an ordered pair?

A Function is a relation in which no two ordered pairs
have the same first element.
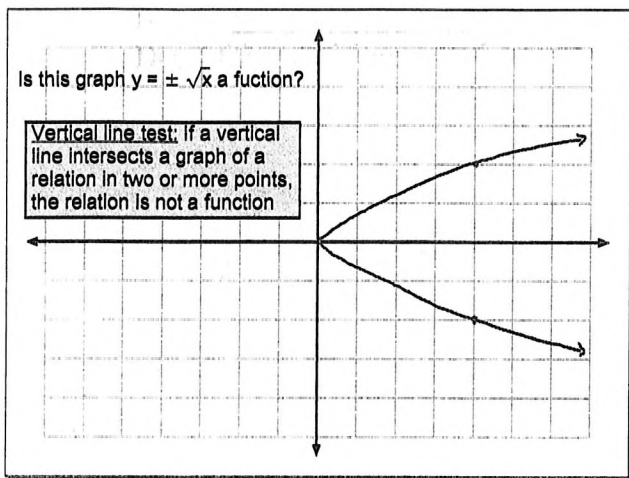
Oct 12-3:38 PM

Sketch: $y = 2x + 3$ Is this a function?

Domain: Is the set of all first elements. (x-values)

Range: Is the set of all second elements. (y-values)

Oct 12-3:47 PM

Are the following functions? Find the domain and range.

a) (2,4) (3,5) (4,6) (5,7)
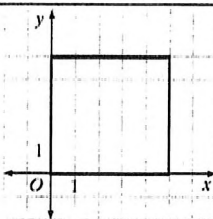
b) (2,1) (2,2) (2,3) (2,4)

c) (3,2) (4,2) (5,2) (6,2)

Oct 12-3:55 PM

8-2

Is this graph $y = \pm \sqrt{x}$ a fuction?

Vertical line test: If a vertical line intersects a graph of a relation in two or more points, the relation is not a function

Oct 12-4:17 PM

The graph of a relation is shown at the right.

a. Determine whether or not the graph represents a function.

b. Find the domain of the graph.

c. Find the range of the graph.

Oct 12-4:11 PM

Find the largest domain of each function:

a. $y = \frac{1}{x + 3}$

b. $y = \frac{x + 1}{x^2 - 2x - 3}$

c. $y = \frac{x}{5x^2 + 1}$

Oct 12-4:30 PM

8-3

Oct 12-4:34 PM

8-4

Name: _____

Day 1 Classwork/Homework

1) Which graph of a relation is *not* a function?

A)

B)

C)

D)
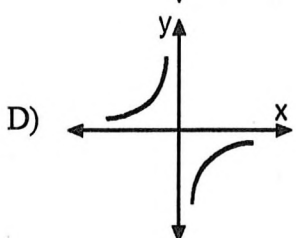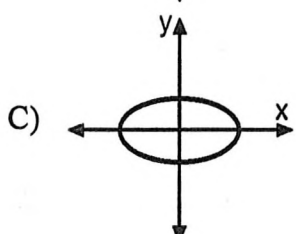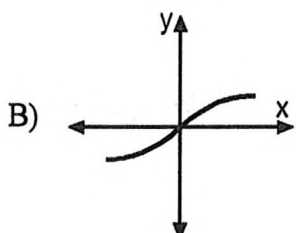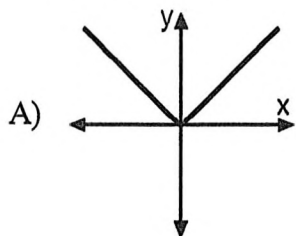
2) For the graph of the relation below,
   (a) state the domain.
   (b) state the range.
   (c) state whether or not the relation is a function.
       [*Justify your answer.*]
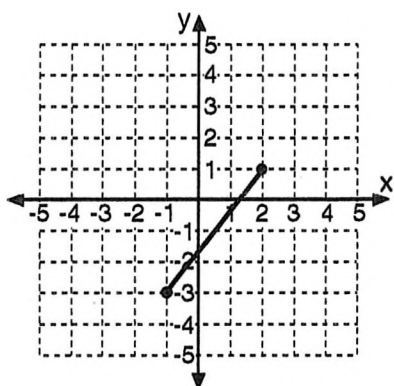
3) The function $f(x) = \dfrac{1}{x-3}$ is defined for *all* real numbers except when x is

A) -3   C) 0

B) 3   D) $-\dfrac{1}{3}$

4) For the graph of the relation below,
   (a) state the domain.
   (b) state the range.
   (c) state whether or not the relation is a function.
       [*Justify your answer.*]
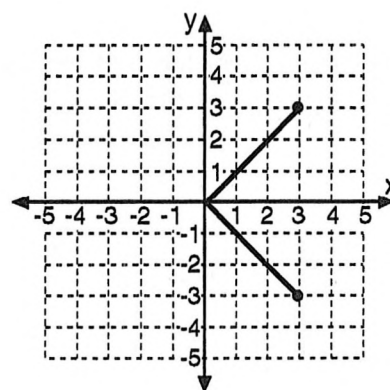
5) Given the relation $R = \{(-2,3), (a,4), (1,9), (0,7)\}$, which replacement for a makes this relation a function?

A) 1   C) 0

B) -2   D) 4

6) Which relation is *not* a function?

A) $\{(1,4), (3,8), (5,16)\}$

B) $\{(1,1), (1,2), (1,3)\}$

C) $\{(1,1), (2,2), (3,3)\}$

D) $\{(1,1), (2,1), (3,1)\}$

8 - 5

7) Which graph of a relation is also a function?

A)

B)

C)
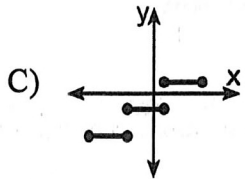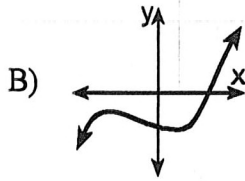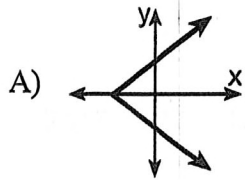
D) all of the above

8) What values of x which would cause the relation $\{(2,7), (4,8), (x,3), (5,6)\}$ *not* to be a function?

9) Is this relation a function? [*Justify your answer.*]

$\{(1,2), (5,8), (3,4), (6,3)\}$

10) Which graph of a relation is also a function?

A)

B)

C)

D)

8 - 6

11) Which graph of a relation is *not* a function?

A)

B)

C)

D)
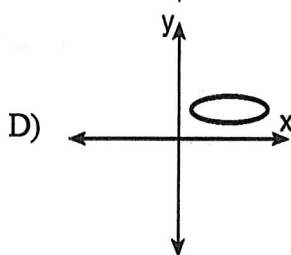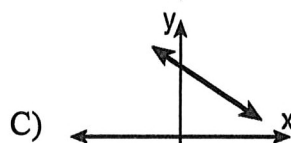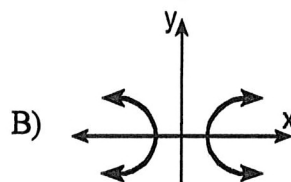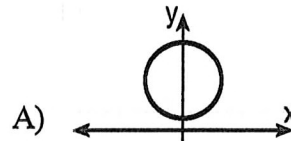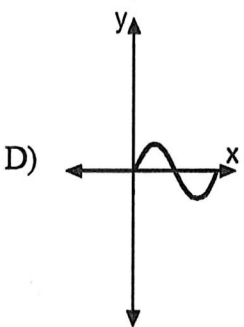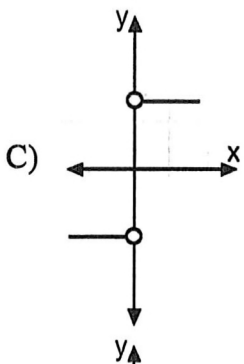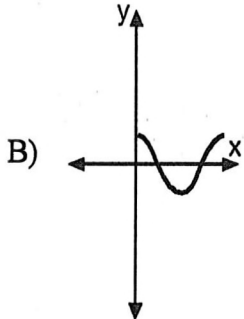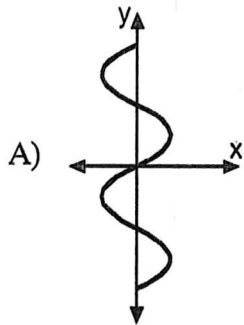
12) The relation defined by the set of ordered pairs {(0,2), (-2,2), (1,4), (0,-1)} is *not* a function. Which of the ordered pairs listed below, if omitted from this relation, will make the resulting set a function.

A) (1,4)   C) (0,-1)
B) (-2,2)   D) (4,1)

13) Is this relation a function? [*Justify your answer.*]

{(-4,2), (-4,1), (-4,0), (-4,-1)}

14) Which graph of a relation is also a function?

A)

B)

C)

D) all of the above

8 - 7
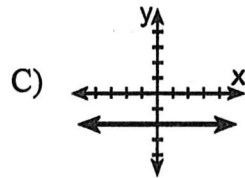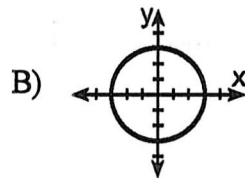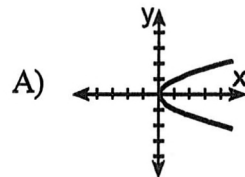
Day 2
Function Notation

$f(x) = 2x + 3$ The function value paired with x is $2x + 3$.

f(x) replaces y =

Oct 13-2:16 PM

If $f(x) = 2x + 3$, evaluate the following:

a) $f(-4)$ b) $f(a + 1)$

c) $f(2x)$ d) $f(x^2)$

Oct 13-2:19 PM

Let f be the set of ordered pairs such that the second element of each pair is 1 more than twice the first.

a) Write $f(x)$ in terms of x.

b) Find $f(7)$.

Oct 13-2:22 PM

8-8

1

Is this a fuction?

Find the following:

a) f(-4)

b) f(0)

c) f(3)

d)f(3/2)

Oct 13-2:37 PM

Let f(x) be defined by the graph above. If
f(−4) + f(2) = f(k), then k is equal to
(A) −3   (B) −1   (C) 0
(D) 1   (E) 3

Oct 13-2:57 PM

15. The graph of the function f is shown at
the right.

Find:

a. f(2)

b. f(−5)

c. f(−2)

d. f$\left(3\frac{1}{2}\right)$

Oct 13-2:25 PM

8- 9
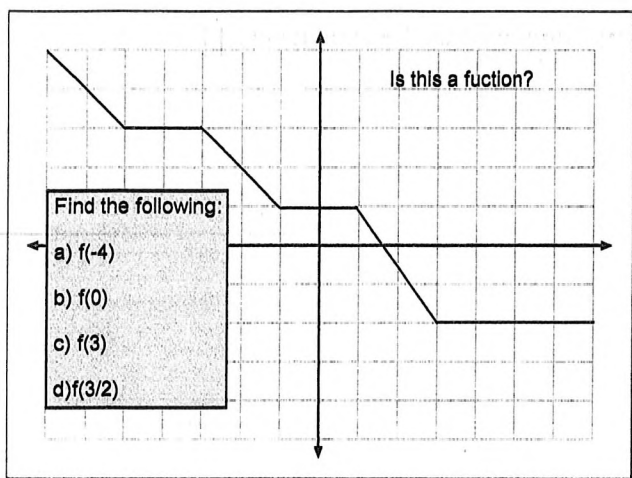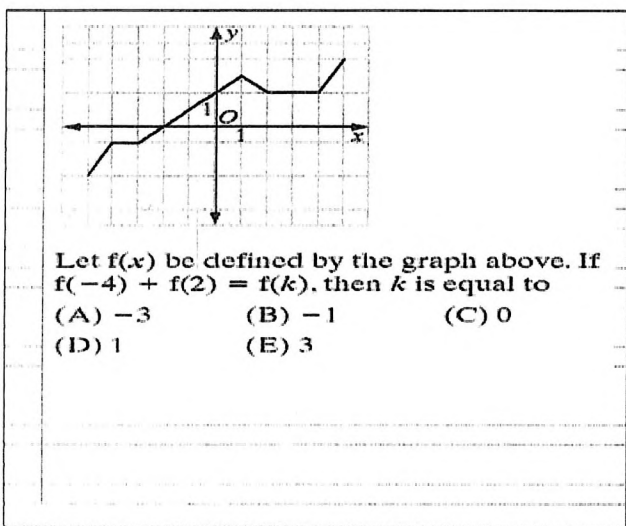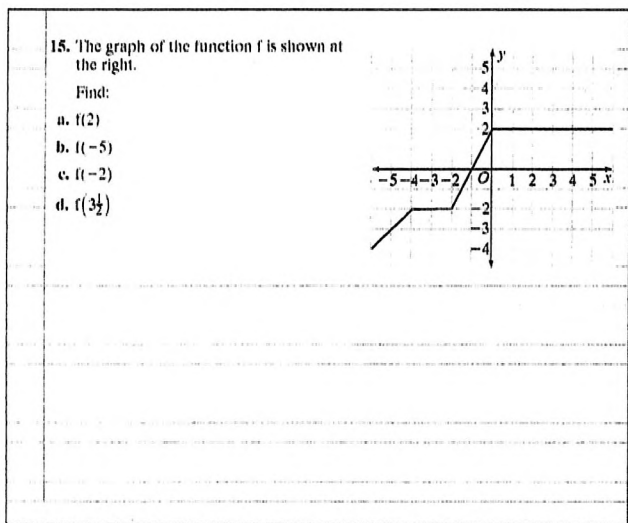
Name: _____

Day 2 Classwork/Homework

1) Given the function $f(x) = (x + 1)^2$, find the value of $f(-2)$.

2) Given the function $g(x) = x^2 - 2x + 1$, find the value of $g(-1)$.

3) If $g(x) = x^{-\frac{3}{2}}$, find $g(4)$.

4) If $f(x) = x^{-\frac{1}{2}}$, find $f(9)$.

5) If $f(x) = \frac{1}{2}x^2$, what does $f(2) - f(-2)$ equal?

   A) -2   C) 0
   B) 2   D) 1

6) Given the function $g(x) = x^2 - 2x + 1$, find the value of $g(1)$.

7) If $f(x) = |x^3 - 3|$, then $f(-1)$ is equivalent to

   A) -2   C) 2
   B) 4   D) 0

8) Given the function $f(x) = 3x - 2$, find the value of $f(0)$.

9) If $f(x) = x^2 + 3x - 5$, find the value of $f(3)$.

10) Given the function $f(x) = 3x - 2$, find the value of $f(-1)$.

11) Given the function $g(x) = x^2 - 2x + 1$, find the value of $g(3)$.

Day 3 Linear Functions and Direct Variation

Recall that a relation is a **function** if no two ordered pairs have the same first element. We can test for a function using the **vertical line test**.

Definition:
A function f is said to be **one-to-one** or (1-1) if no two ordered pairs have the same *second* element.
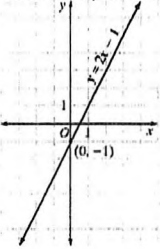
Write down 4 readable points, is this set of ordered pairs one-to-one?

Can you think of a way to decide if a function is 1-1 just by looking at the graph?

HORIZONTAL LINE TEST

Oct 14-9:58 AM

Determine if each of the following functions is one-to-one.

a. $y = 4 - 3x$ **b.** $y = x^2$ **c.** $y = |x + 3|$

Oct 14-10:48 AM

Direct Variation
When the ratio of two variables is a constant, we say that the variables are directly proportional or that the variables vary directly. Every direct variation of two variables is a linear function that is one-to-one.

ex. Jacob can type 55 words per minute.

  a. Write a function that shows the relationship between the number of words typed, w, and the number of minutes spent typing, m.

  b. Is the function one-to-one?

  c. If Jacob types for no more than 2 hours at a time, what are the domain and range of the function?

  a. "Words per minute" can be written as $\frac{words}{minutes} = \frac{w}{m} = 55$. This is a direct variation function that can be written as $[(m, w) : w = 55m]$.

  b. The function is a linear function and every linear function is one-to-one.

  c. If Jacob types no more than 2 hours, that is, 120 minutes, at a time, the domain is $0 \leq m \leq 120$ and the range is $0 \leq w \leq 6,600$.

Oct 14-11:08 AM

8 - 11

1

Types of Functions

Constant Function	Linear Function
$y = a$, with a a constant. Example: $y = 3$	$y = mx + b$, with m and b constants. Example: $y = 2x + 3$

The range is {3}.
The function is not onto.

The range is the set of real numbers.
The function is onto.

Quadratic Function	Absolute Value Function
$y = ax^2 + bx + c$ with a, b, and c constants and $a \neq 0$. Example: $y = x^2 - 4x + 3$	$y = a + |bx + c|$ with a, b, and c constants. Example: $y = -2 + |x + 1|$

The range is the set of real numbers ≥ -1.
The function is not onto.

The range is the set of real numbers ≥ -2.
The function is not onto.

Square Root Function

$y = \sqrt{ax + b}$, with $ax + b \geq 0$.

- The domain is the set of real numbers greater than or equal to $-\frac{b}{a}$.
- The range is the set of non-negative real numbers.

Example: $y = \sqrt{x}$

This is a function from the set of non-negative real numbers to the set of non-negative real numbers.

The range is the set of non-negative real numbers.
The function is onto.

Oct 15-11:40 AM

8-12

Day 3 Classwork

1) If $f(x) = 4x^0 + (4x)^{-1}$, what is the value of $f(4)$?

2) If $f(x) = (x^{-x} - x^0 + 2^x)$, then $f(3)$ is equal to

3) What is the domain of the function $f(x) = \dfrac{3x^2}{x^2 - 49}$?

4) If $f(x) = \dfrac{1}{\sqrt{2x-4}}$, the domain of $f(x)$ is

5) Which equation does *not* represent a function?

[A] $y = x^2 + 5x$ [B] $x = \pi$

[C] $y = |x|$ [D] $y = 4$

8-13

1

Day 4 Composition of Functions

Composite Function - is the combining of two functions f and g, to produce a new function symbolized f•g.

f • g is read "f following g."

f(g(x))

Are f and g functions?

f
0 —— 5
1 —— 6
2 —— 7

g
3 —— 0
4 —— 1
5 —— 2
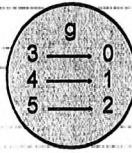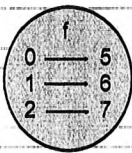
Evaluate f • g:

Whats comes first??

g   f
3 —— 0 —— 5
4 —— 1 —— 6
5 —— 2 —— 7

f • g
3 —— 5
4 —— 6
5 —— 7

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f • g
3 —— 5
4 —— 6
5 —— 7
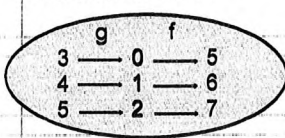
Find the following:

(f • g)(3)

(f • g)(4)

(f • g)(5)

In order for the composite function f • g to exist, each member of the range of g must be a member of the domain of f.

8-14

1

$f(x) = x^2$ $g(x) = 3x$ $h(x) = x - 1$

Find:

a) $f \circ g \,(1)$

b) $h \circ f \,(3)$

c) $g(f(-2))$

Name: _____

Day 4 CW/HW

1) If $f(x) = x - 3$ and $g(x) = x^3$, then $f(g(3))$ is
   A) 24   C) 0
   B) 30   D) 6

2) If $f(x) = 2x$ and $g(x) = x - 4$, what is the value of $f(g(3))$?
   A) -6   C) 6
   B) -2   D) 2

3) If $f(x) = 2x$ and $g(x) = x - 4$, what is the value of $g(f(3))$?
   A) 2   C) -6
   B) 6   D) -2

4) If $g(x) = x + 3$ and $f(x) = x^2 - 2$, find the value of $f(g(-3))$.

5) If $f(x) = x - 3$ and $g(x) = x^2$, what is the value of $(f \circ g)(2)$?

6) If $f(x) = 3x - 1$ and $g(x) = x^2 + 1$, evaluate $(g \circ f)(-1)$.

7) If $f(x) = 3x$ and $g(x) = 7x - 1$, what is $(f \circ g)(4)$?

8) If $g(x) = x + 3$ and $f(x) = x^2 - 2$, find the value of $g(f(-1))$.

9) If $g(x) = x + 3$ and $f(x) = x^2 - 2$, find the value of $f(g(3))$.

10) If $g(x) = x + 3$ and $f(x) = x^2 - 2$, find the value of $g(f(3))$.

11) If $f(x) = 3x^2$ and $g(x) = \sqrt{2x}$, what is the value of $(f \circ g)(8)$?
    A) 144   C) 48
    B) 16   D) $8\sqrt{6}$

12) If $g(x) = |x - 3|$ and $h(x) = x^2 + 2x$, what is the value of $h(g(2.9))$?
    A) -0.99   C) 0.99
    B) 0.21   D) -0.21

8 - 16

Day 5 Inverse Functions

f -1 is the inverse of f under composition

f: x → y   f-1: y → x

ex. f = {(1,6) (2,7) (3,0)}   find f -1

ex. g = {(4,8) (5,8) (6,9)}   find g -1

Oct 25-9:20 PM

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ex. f = {(1,6) (2,7) (3,0)}   find f-1 = {(6,1) (7,2) (0,3)}

ex. g = {(4,8) (5,8) (6,9)}   find g-1= {(8,4) (5,8) (9,6)}

Is f a function? Is f -1 a function?

Is g a function? Is g-1 a function?

How can we tell if a function's inverse is going to be a function?

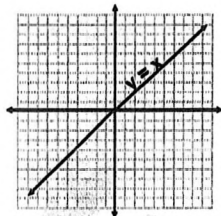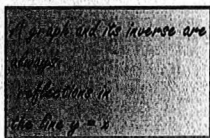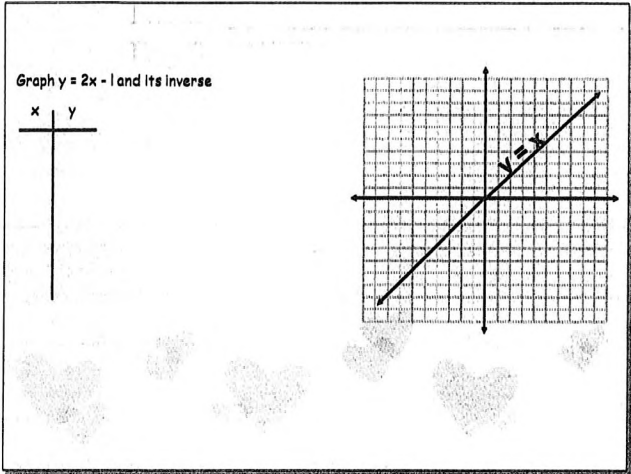Oct 25-9:31 PM

Ways to find the Inverse:
1. ordered pairs- switch x & y
h(x) = (14,32) (-21, 18) (49, 0)   h-1(x) =

2. coordinate graph-switch x & y and plot new points

A graph and its inverse are
always
reflections in
the line y=x

y=x

Oct 25-9:43 PM

Graph y = 2x - 1 and its inverse

x	y

$y = x$

Oct 25-9:55 PM

3. Rule of the inverse

ex. find the inverse of f(x) = 2x - 4

The composition of a function and its inverse is the identity function J(x) = x

Oct 25-10:00 PM

Find the inverse of the following:

f(x) = -3x + 5 $g(x) = \dfrac{1x}{3} + 10$ $h(x) = \dfrac{x + 2}{7}$
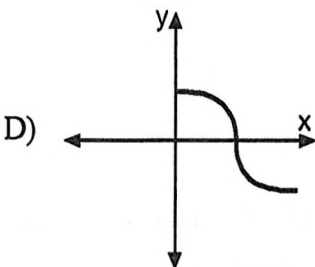
Oct 26-8:36 AM

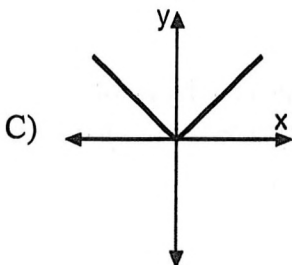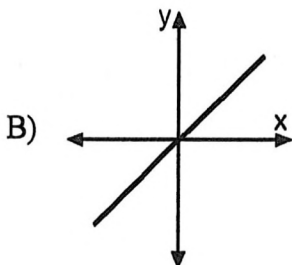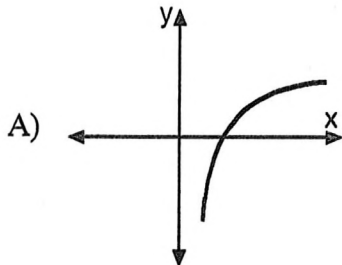8-18

2

Let $f(x) = 8x$. The graph of $g(x)$ is the graph of f shifted 4 units down and 6 units to the right. Write an expression for $g(x)$. And find $g^{-1}(x)$.

Oct 26-8:45 AM

8-19

Name: _____

Day 5 CW/HW

1) Which function is *not* one to one?

A)

B)

C)

D)

2) Given: set $A = \{(1,2), (2,3), (3,4), (4,5)\}$

If the inverse of the set is A^{-1}, which statement is true?

A) A and A^{-1} are not functions.

B) A and A^{-1} are functions.

C) A is not a function and A^{-1} is a function.

D) A is a function and A^{-1} is not a function.

3) Which equation defines a function whose inverse is *not* a function?

A) $y = |x|$ C) $y = -x$

B) $y = 2^x$ D) $y = 3x + 2$

4) What is the inverse of the equation $y = 3x - 2$?

A) $y = 3x + 2$ C) $y = \frac{x+2}{3}$

B) $y = x$ D) $y = 2x - 3$

5) What is the inverse of the equation $y = 3x$?

A) $x = \frac{y}{3}$ C) $y = 3$

B) $y = \frac{1}{3}x$ D) $x = 3$

6) What is the inverse of the function $y = 2x - 5$?

A) $y = 2x + 5$ C) $y = \frac{1}{2}(x + 5)$

B) $y = 5 - 2x$ D) $y = \frac{1}{2}(x - 5)$

7) The inverse function of $\{(2,6), (-3,4), (7,-5)\}$ is

A) $\{(2,-6), (-3,-4), (7,5)\}$

B) $\{(6,2), (4,-3), (-5,7)\}$

C) $\{(-2,6), (3,4), (-7,-5)\}$

D) $\{(-6,-2), (-4,3), (5,7)\}$

8-20

8) If (-3, 1) is in the function $f(x)$, which of the following points will be in the function $f^{-1}(x)$?

A) (-1,3)   C) (1,-3)

B) (3,-1)   D) (3,1)

9) The inverse of the function $y = x^2 + 9$, $x \geq 0$ is

A) $y = \sqrt{x - 9}$ C) $y = -\sqrt{x + 9}$

B) $y = \pm\sqrt{x + 9}$ D) $y = \sqrt{x + 9}$

10) Write the inverse of the given function:

$$\{(a,b), (c,d), (e,f), (g,h)\}$$

11) Find f^{-1}, the inverse of the given function:

$$f(x) = \{(8,2), (1,7), (4,3), (5,1)\}$$

Questions 12 through 15 refer to the following:

For the given relation(s),
   (a) state the inverse.
   (b) state whether or not the inverse is a function.
   [*Justify your answer.*]

12) $\{(3,2), (4,3), (5,8), (6,-1)\}$

13) $\{(1,2), (1,3), (1,4), (1,5)\}$

14) $\{(1,1), (2,1), (3,1), (4,1)\}$

15) $\{(x,y) \mid y = 6x - 2\}$

16) The graph of any function and the graph of its inverse are symmetric with respect to the

A) graph of the equation $y = -x$

B) graph of the equation $y = x$

C) y-axis

D) x-axis

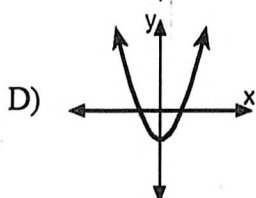17) Given the function $f(x) = \{(3,1), (2,6), (-3,5)\}$. Find f^{-1}, the inverse of function f.
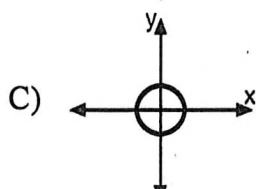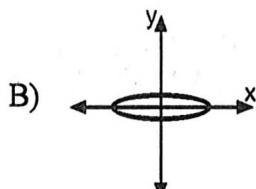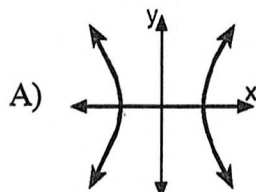
18) Write the inverse of the given function:

$$\{(5,3), (-2,4), (7,-2)\}$$

Name: _____

Review for Function Test

1) Which graph illustrates a quadratic relation whose domain is *all* real numbers?

A)

B)

C)

D)

2) The function $f(x) = \dfrac{1}{x-3}$ is defined for *all* real numbers except when x is

A) -3   C) 3

B) 0   D) $-\dfrac{1}{3}$

3) Given the real valued function $f(x) = \dfrac{1}{\sqrt{x-3}}$, which number is in the domain of $f(x)$?

A) -1   C) 2

B) 7   D) 3

4) The domain of $f(x) = x^2 + 2x + 1$ is $-3 \le x \le 3$. The *largest* value in the range of $f(x)$ is

A) 20   C) 16

B) 4   D) 3

5) For which value of x is the function
$g(x) = \dfrac{2x^2 + 3x + 1}{x - 1}$ undefined?

6) Given the function $g(x) = 3x - 3$ with a domain of $\{x \mid -2 \le x \le 4\}$, find the range.
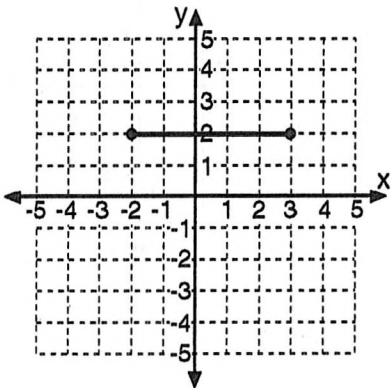
7) For the graph of the relation below,
   (a) state the domain.
   (b) state the range.
   (c) state whether or not the relation is a function.
   [*Justify your answer.*]

8 - 22
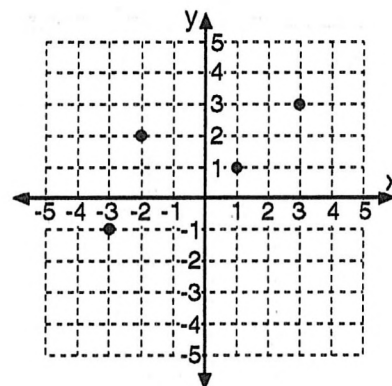
8) For the graph of the relation below,
   (a) state the domain.
   (b) state the range.
   (c) state whether or not the relation is a function.
   [*Justify your answer.*]
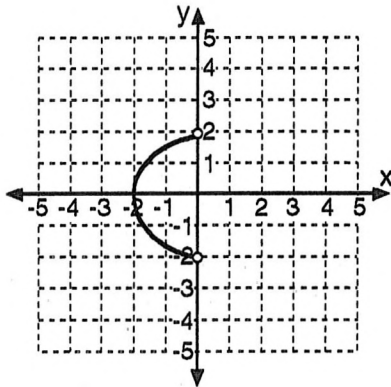
9) For the graph of the relation below,
   (a) state the domain.
   (b) state the range.
   (c) state whether or not the relation is a function.
   [*Justify your answer.*]
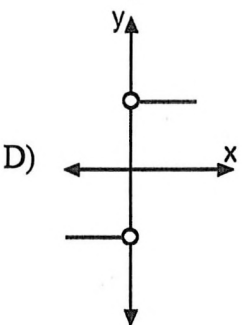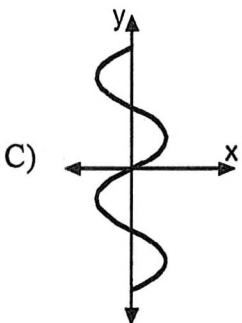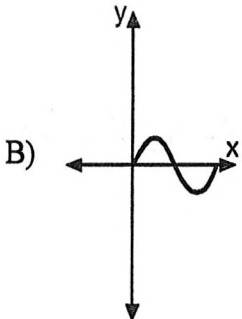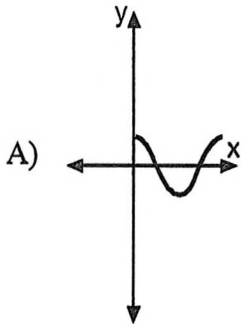

10) Which graph of a relation is also a function?

A)

B)

C)

D)

11) Which graph of a relation is also a function?

A)

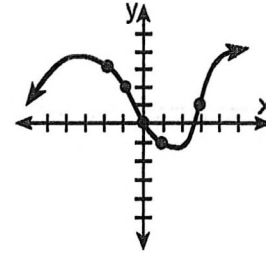B)

C)

D) all of the above

8 - 23
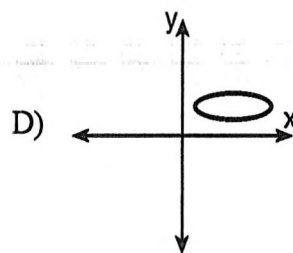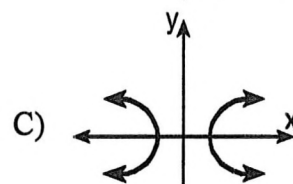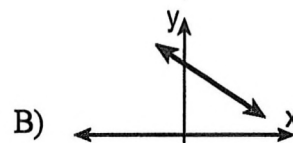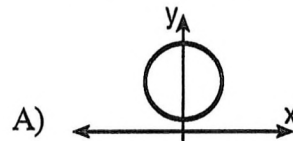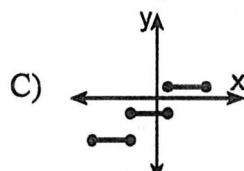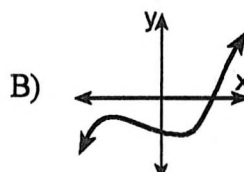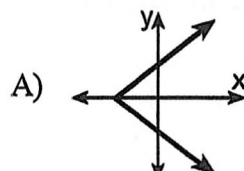
12) Which graph of a relation is *not* a function?

A)

B)

C)

D)

13) The relation defined by the set of ordered pairs {(0,2), (-2,2), (1,4), (0,-1)} is *not* a function. Which of the ordered pairs listed below, if omitted from this relation, will make the resulting set a function.

A) (1,4)   C) (-2,2)
B) (0,-1)   D) (4,1)

14) If the graph below is the graph of $y = f(x)$ what is the value of $f(1)$?


A) -1   C) -2
B) 2   D) 1

15) Given the function $f(x) = (x + 1)^2$, find the value of $f(-2)$.

16) Given the function $f(x) = 3x - 2$, find the value of $f(-1)$.

17) If $f(x) = x^2 + 3x - 5$, find the value of $f(3)$.

18) If $g(x) = \sqrt{x}$ and $h(x) = x^3 - 1$, what is $g(h(4))$?

A) 5   C) $\sqrt{63}$
B) $\sqrt{11}$ D) 7

19) If $f(x) = 2x$ and $g(x) = x - 4$, what is the value of $f(g(3))$?

A) 6   C) -2
B) 2   D) -6

20) If $f(x) = 5 - 2x$ and $g(x) = |x - 3|$, what is the value of $f(g(3))$?

A) -1   C) 5
B) 7   D) 1

8 - 24

21) If $g(x) = |x - 3|$ and $h(x) = x^2 + 2x$, what the value of $h(g(1.4))$?

A) 6.40 C) 5.76

B) 0.46 D) -0.46

22) Which function is *not* one to one?

A)

B)

C)

D)
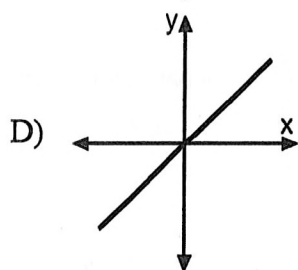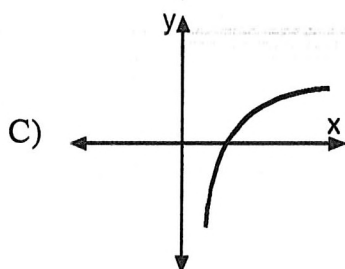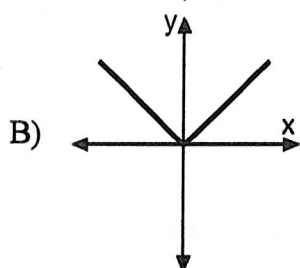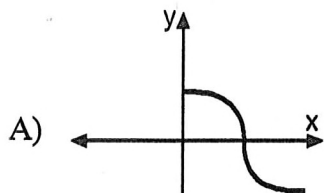
23) What is the inverse of the equation $y = 3x + 2$?

A) $x = \frac{1}{3}y + \frac{2}{3}$ C) $x = 3y + 2$

B) $y = \frac{1}{3}x - 2$ D) $3y = x + 2$

24) What is the inverse of the function $x + 2y + 3 = 0$?

A) $2x - y + 3 = 0$

B) $y = -2x - 3$

C) $2y + x + 3 = 0$

D) $y = -\frac{1}{2}x - \frac{3}{2}$

25) The inverse function of $\{(2,6), (-3,4), (7,-5)\}$ is

A) $\{(6,2), (4,-3), (-5,7)\}$

B) $\{(2,-6), (-3,-4), (7,5)\}$

C) $\{(-6,-2), (-4,3), (5,7)\}$

D) $\{(-2,6), (3,4), (-7,-5)\}$

8-25