Complex Numbers

Day 1
Imaginary Numbers

Show video on Complex Numbers -- Electricity

Jul 7-8:21 AM

The Imaginary Unit is defined as

$$i = \sqrt{-1}$$

The reason for the name *"imaginary"* numbers is that when these numbers were first proposed several hundred years ago, people could not *"imagine"* such a number.

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Imaginary numbers occur when a quadratic equation has no roots in the set of real numbers.

$$x^2 + 1 = 0$$

$$x^2 = -1$$

$$\sqrt{x^2} = \sqrt{-1}$$

$$x = +\sqrt{-1} \text{ or } x = -\sqrt{-1}$$

$$* \; i = \sqrt{-1} \quad \text{or} \; -i = -\sqrt{-1}$$

Jul 7-8:17 AM

7-2

A pure imaginary number can be written in *bi* form where *b* is a *real* number and *i* is $\sqrt{-1}$

Examples: pure imaginary numbers $2i$, $-5i$, $3i\sqrt{3}$, $\frac{3}{2}i$

A **complex number** is any number that can be written in the standard form **a + b*i***, where a and b are real numbers and *i* is the imaginary unit.

A complex number is a real number a, or a pure imaginary number b*i*, or the sum of both. Note these examples of complex numbers written in standard a + b*i* form: $2 + 3i$, $-5 + 0i$.

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Complex Numbers standard a + b*i* form	a	b*i*
$7 + 2i$	7	$2i$
$1 - 5i$	1	$-5i$
$8i$	0	$8i$
$\frac{-2+3i}{5} \Rightarrow \frac{-2}{5} + \frac{3i}{5}$	$\frac{-2}{5}$	$\frac{3i}{5}$

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Imaginary Numbers Day 1

A PURE IMAGINARY NUMBER is any number that can be expressed in the form b*i*, where b is the real number not equal to zero and *i* is the imaginary unit $\sqrt{-1}$.

Examples:

1) $\sqrt{-25}$ 2) $\sqrt{-4}$ 3) $\sqrt{-36}$ 4) $\sqrt{-100}$

5) $\sqrt{-8}$ 6) $\sqrt{-48}$ 7) $\sqrt{-20}$ 8) $\sqrt{-75}$

Simplify in terms of *i* and combine:

1) $\sqrt{-16}+\sqrt{-49}$ 2) $\sqrt{-25}-\sqrt{-144}$ 3) $\sqrt{-9}+\sqrt{-100}$

4) $5\sqrt{-36}+2\sqrt{-81}$ 5) $8\sqrt{-100}-4\sqrt{-64}$ 6) $5\sqrt{-25}+6\sqrt{-9}$

7) $\sqrt{-72}+\sqrt{-32}$ 8) $\sqrt{-18}+\sqrt{-75}$ 9) $\sqrt{-63}-\sqrt{-12}$

10) $\sqrt{-49}-2\sqrt{-4}$ 11) $5\sqrt{-27}-3\sqrt{-12}$ 12) $4\sqrt{-20}+\frac{1}{5}\sqrt{-125}$

DAY 2

Find the value of the following: Remember $i =$

$i^0 =$

$i^1 =$

$i^2 =$

$i^3 =$

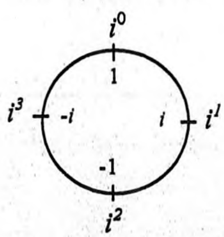
$i^4 =$

$i^5 =$

Nov 6-11:06 AM

Cyclic Nature of the Powers of *i*

To be *cyclic* means to be repetitive in nature. When the imaginary unit, *i*, is raised to increasingly larger powers, it creates a cyclic pattern.



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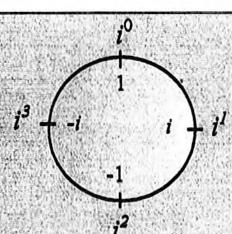
Write each number in simplest form:



1) i^{23}

2) i^{37}

3) i^{152}

4) i^{5000}

[redacted]

Nov 6-11:12 AM

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Multiplying two complex numbers is accomplished in a manner similar to multiplying two binomials; you can use the distributive or FOIL process of multiplication.

1). Multiply 2i (4 + 3i) and express your answer in $a + bi$ form.

Oct 22-8:01 AM

2). Multiply (4 - 2i)(5 + i) and express your answer in a + bi form.

3). Multiply (5 - 7i)(-2 + 3i) and express your answer in a + bi form.

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5). Multiply (3 - 2i)(3 + 2i) .

The **conjugate** of a complex number **a + b*i*** is the complex number **a - b*i***.
For example, the conjugate of **4 + 2*i*** is **4 - 2*i***.
(Notice that only the sign of the **b*i*** term is changed.)

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4). Simplify $(1 - 5i)^2$

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6). Express the product of $(1/3 - 1/2i)(12 + 3i)$ in $a + bi$ form.

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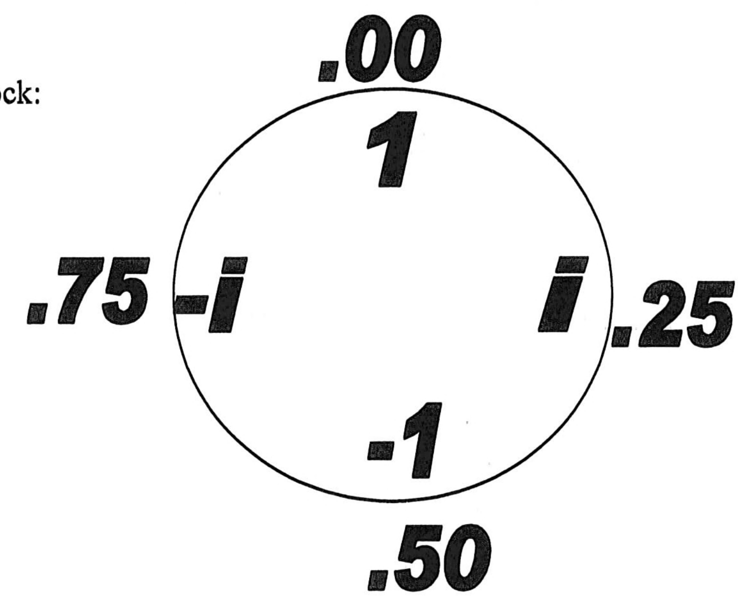
Name________________________ Day 2 Homework Date______________

Powers of i

$i^0=1$	$i^1=i$	$i^2=-1$	$i^3=-i$
$i^4=1$	$i^5=i$	$i^6=-1$	$i^7=-i$
$i^8=1$	$i^9=i$	$i^{10}=-1$	$i^{11}=-i$

ect……

What about i^{2005} ? Use the i clock:



$2005 \div 4 = 501.\mathbf{25} = i$

Try

i^{82} i^{2007} i^{2008} i^{78} i^{50} i^{145673}

Simplify in terms of i:

1) $\sqrt{-9} \bullet \sqrt{-8}$

2) $\sqrt{-25} \bullet \sqrt{-36}$

3) $\sqrt{-20} \bullet \sqrt{-4}$

4) $i(4+i) - 4i$

5) $i(9-2i)-2$

6) $i(1+i)+i^3$

7) $5(2-i)-5i^3$

8) $8+i(8-i)$

9) $3i^3-3(2-i)$

10) $6i^3+2(7+3i)$

11) $i^3 + i(10-i)$

12) $i^3-i(5-i)$

Name________________________________ Date____________

Day 3

Classwork

Multiplication of complex numbers

1) $6i(5i)$

2) $6i(10-2i)$

3) $(3+ i) (4 +i)$

4) $(5-i) (3+i)$

5) $(2+i)^2 -3$

6) $(10+2i) (10-2i)$

7) $(1+3i)(1-4i)$

8) $(2 + \sqrt{-49})^2$

9) $(3 + 2i)(4-i)$

10) $(3+7i) (1-2i)$

11) $6i^3+ 2(7+3i)$

12) $i^3+i(10-i)$

13) $(1+i)^2- 2i$

14) $(2+0i)(8-5i)$

15) $(8-i)^2$

16) $(5-4i)^2$

Name________________________________ Date____________

Day 3 Homework

Find the product in a+bi form:

1) $(3 + i)(4 + i)$

2) $(3 - i)(2 - i)$

3) $(5 + i)(6 - i)$

4) $(4 - 5i)(2 + i)$

5) $(8 - 5i)(3 - i)$

6) $(2 - i)(4 + 2i)$

7) $(8 + \sqrt{-25})(2 + \sqrt{-1})$

8) $(3 - \sqrt{-49})(2 + \sqrt{-16})$

9) $(5 - \sqrt{-36})(2 - \sqrt{-100})$

10) $(6 + i)^2$

11) $(2 + 5i)^2$

13) $(5 - 4i)^2$

14) $(1 + i)^3$

15) $(2 - 2i)^3$

16) $(3 + 5i)(3 - 5i)$

17) $(7 - 10i)(7 + 10i)$

18) $3i^3 - 3(2 - i)$

19) $6i^3 + 2(7 + 3i)$

20) $i(1 + i) + i^3$

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Day 4

1. Simplify: $\sqrt{-112}$

2. Simplify: i^{4321}

3. Simplify: $(i^{27})^3$

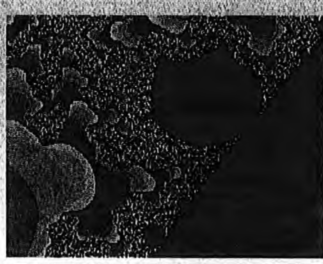
4. Simplify: $i^{17} \cdot i^{23}$

5. Find the difference: $(2 - 5i) - (-7 + 2i)$

6. Find the product in simplest a + bi form:
$(4 - 3i)(1 + 2i)$

Nov 10-10:32 AM

Multiplicative Inverse and Dividing Complex Numbers



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The multiplicative inverse of a complex number a + bi is 1 ÷ a + bi.

(Use the conjugate to rationalize the denominator.)

1). Find the multiplicative inverse of 3 + 2i.

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2). Find the multiplicative inverse of 4 - 5i.

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When dividing two complex numbers,

1. Write the problem in fractional form,

2. Rationalize the denominator by multiplying the numerator and the denominator by the conjugate of the denominator.

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1). Express the results in a + bi form:

a). (4 - 6i) ÷ (1 - i)

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b). $(1 + 2i) \div (2 + 3i)$

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c). $(5 + i) \div (3 - 4i)$

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Day 4
Class work/Homework

Find the multiplicative inverse of 3-*i*.

$\frac{1}{3-i}$ **You cannot have an *i* in the denominator**

$$\frac{1}{3-i}\bullet\frac{3+i}{3+i}=\frac{3+i}{(3-i)(3+i)}=\frac{3+i}{9+3i-3i-i^2}=\frac{3+i}{9-i^2}=\frac{3+i}{9-(-1)}=\frac{3+i}{10}\ or\ \frac{3}{10}+\frac{i}{10}$$

Find the multiplicative inverse of each:

1) 3+*i*

2) 4+3*i*

3) 9-2*i*

4) $5-3i$

5) $7-4i$

6) $3-i\sqrt{2}$

7) $\sqrt{3}+i$

8) $5-i\sqrt{5}$

Name______________________________ Date_________

Day 5
Class work

Multiplicative Inverse:

You cannot have an *i* in the denominator. You have to **RATIONALIZE** the denominator. Multiply by the **conjugate**

Find the multiplicative inverse of 3-i.

$\frac{1}{3-i}$ **You cannot have an i in the denominator**

$$\frac{1}{3-i} \bullet \frac{3+i}{3+i} = \frac{3+i}{(3-i)(3+i)} = \frac{3+i}{9+3i-3i-i^2} = \frac{3+i}{9-i^2} = \frac{3+i}{9-(-1)} = \frac{3+i}{10} \text{ or } \frac{3}{10} + \frac{i}{10}$$

Find the multiplicative inverse of each:

1) 6-i

2) 1+5i

3) 4-3i

4) 4-4i

5) 5+5i

6) 8-4i

7) $\sqrt{5} + i$

8) $2 - i\sqrt{3}$

Name________________________ Date______________

Day 5

Homework

DIVIDING COMPLEX NUMBERS

1) $(8+i) \div (2-i)$ Check:

2) $(3+12i) \div (4-i)$ Check:

3) $(4-2i) \div (1+i)$ Check:

4) $(3-i) \div (2+i)$ Check:

5) $\dfrac{5-3i}{1-i}$ Check:

6) $\dfrac{1-3i}{2-7i}$ Check:

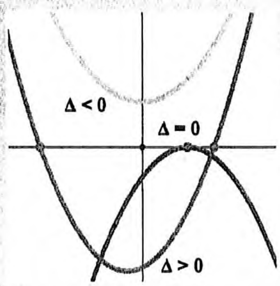
7) $\dfrac{1+3i}{2+4i}$ Check:

8) $\dfrac{3-5i}{i}$ Check:

9) $\dfrac{7+2i}{4i}$ Check:

Day 6

Solving Quadratic Equations



Oct 23-7:39 AM

Find the solution set for Solve for x: $x^2 + x + 2 = 0$

The solutions of some quadratic equations, $ax^2 + bx + c = 0$ ($a \neq 0$), are not rational, and cannot be obtained by factoring. For such equations, the most common method of solution is the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Note: The quadratic formula can be used to solve ANY quadratic equation, even those that can be factored. Be sure you know this very useful formula!!

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Find the solution set for $x^2 + x + 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Oct 23-7:39 AM

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Find the solution set for $x^2 + 4x + 13 = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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Day 6 Class work/ Homework

Quadratic formula with imaginary answers

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1) $X^2-2X+10=0$

2) $X^2-8X+17=0$

3) $\frac{X^2}{2}+3X+5=0$

4) $2X^2+72=0$

5) $9X^2-6X+2=0$

6) $X^2+36=7-10X$

7) $4X(X+5)+29=0$

8) $X^2+20X=2X-86$

9) $13=4X(3-X)$

10) $\frac{X-2}{X}=\frac{X+27}{17}$

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Day 7

Class work / homework

Quadratic formula with imaginary answers

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1) $X^2-4X+5=0$

2) $X^2-6X+25=0$

3) $X^2+26=2X$

4) $X^2+29=10x$

5) $X^2=8X-25$

6) $3X^2=6(X-1)$

7) $4X(X-2)+5=0$

8) $\frac{X-4}{10}=\frac{X-5}{X}$

Name________________________________ Day 8 Date____________

Review

Simplify:

1) $\sqrt{-48}$

2) $\sqrt{-9}+\sqrt{-100}$

3) $5\sqrt{-36}+2\sqrt{-81}$

4) $5\sqrt{-27}-3\sqrt{-12}$

Use i clock:

i^{78} i^{50} i^{145673}

simplify: 1) $i(1+i)+i^3$

Multiply (foil)

1) (3+7i) (1-2i)

2) $(5-\sqrt{-36})(2-\sqrt{-100})$

Multiplicative inverse

1) $1+7i$

Dividing complex numbers

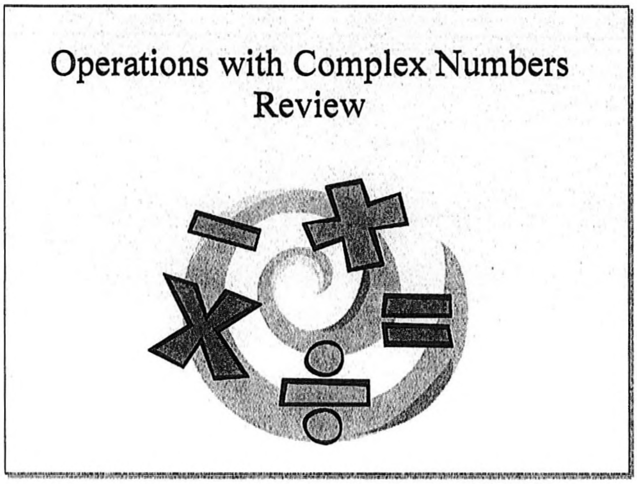
1) $(3-i) \div (2+i)$

Check:

Quadratic formula with imaginary answers

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1) $X^2-4X+5=0$



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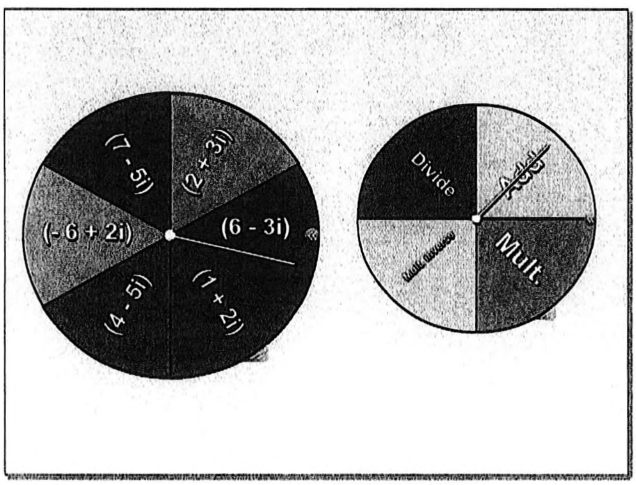

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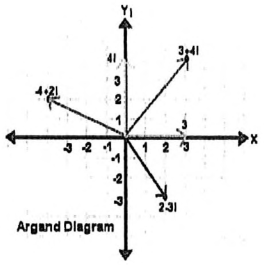
Graphing Complex numbers



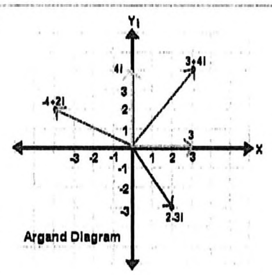
Apr 19-9:00 AM

In the Argand diagram, a complex number $a + bi$ is the point (a,b) or the vector from the origin to the point (a,b).

Graph the complex numbers:

1. $3 + 4i$ $(3,4)$
2. $2 - 3i$ $(2,-3)$
3. $-4 + 2i$ $(-4,2)$
4. 3 (which is really $3 + 0i$) $(3,0)$
5. $4i$ (which is really $0 + 4i$)

The complex number is represented by the point, or by the vector from the origin to the point.



Apr 19-9:01 AM

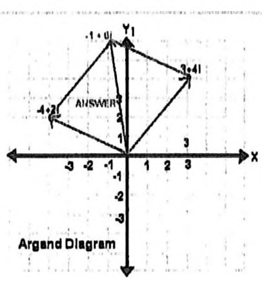
Add $3 + 4i$ and $-4 + 2i$ graphically.

Graph the two complex numbers $3 + 4i$ and $-4 + 2i$ as vectors.

Create a parallelogram using these two vectors as adjacent sides.

The answer to the addition is the vector forming the diagonal of the parallelogram (read from the origin).

This new vector is called the resultant vector.



Apr 19-9:02 AM

Subtract 3 + 4*i* from -2 + 2*i*

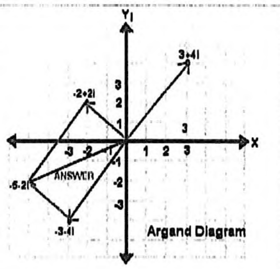
Subtraction is the process of adding the additive inverse.

$(-2 + 2i) - (3 + 4i)$
$= (-2 + 2i) + (-3 - 4i)$
$= (-5 - 2i)$

Graph the two complex numbers as vectors.

Graph the additive inverse of the number being subtracted.

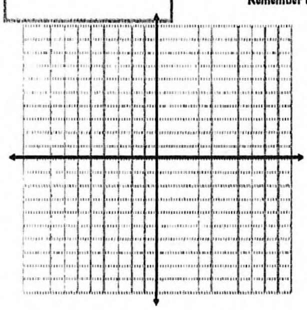
Create a parallelogram using the first number and the additive inverse. The answer is the vector forming the diagonal of the parallelogram.

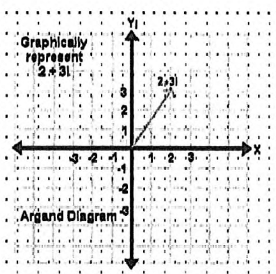


Apr 19-9:02 AM

1. Represent the complex number 2 + 3i graphically.

Remember to draw the vector from the origin.





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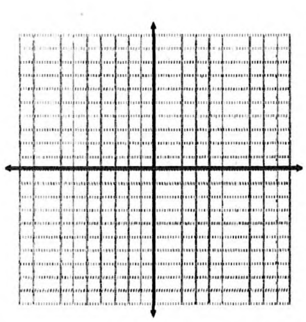
2. Add graphically:
(-2+4i) and (4+i)



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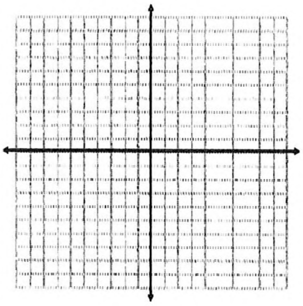
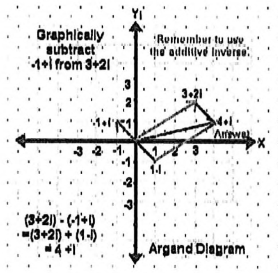
3. Graphically subtract
(-1+i) from (3+2i)

Better! Remember from the origin.

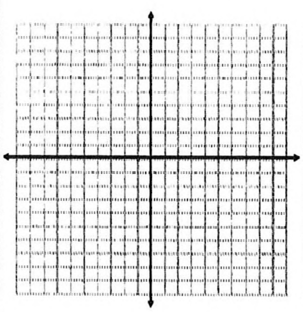




Apr 19-9:08 AM

4. Graphically add:
(2+i) and (4+2i)

Oh, no!! Now what should I do!



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