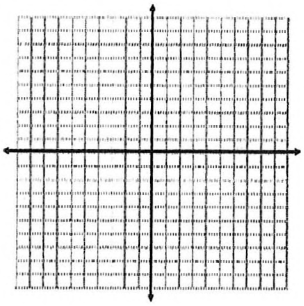
Logs

- look at $y = 2^x$

Day 1

- make a table to values from $-3 \leq x \leq 3$
- plot the points
- plot the inverse



Apr 5-1:50 PM

Logger Dan

$\text{Log}_D A = N$

$D^N = A$

Apr 6-7:21 AM

$\text{Log}_D A = N$ **$D^N = A$**

Write each expression in exponential form:

a) $\log_8 2 = 1/3$ b) $7 = \log_2 128$

Write each expression in logarithmic form:

a) $12^0 = 1$ b) $4^{-2} = 0.0625$

Apr 6-7:27 AM

$\log_D A = N$ **$D^N = A$**

Solve:

1) $\log_8 8 = x$

2) $\log_5 625 = x$

3) $\log_b 27 = 3$

4) $\log_b 64 = 6$

5) $\log_{25} x = -4$

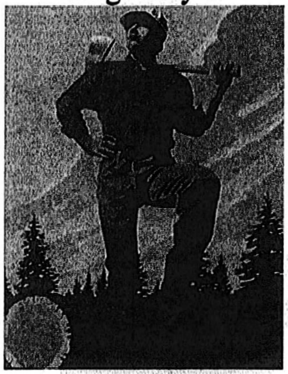
6) $\log_{100} x = -1/2$

Apr 6-7:27 AM

10-3

Name____ Date______________

Logs Day 1



Logger DAN

Logger DAN cut his finger and showed his DNA

The logarithmic equation:

$Log_D A=N$

The Exponential equation:

$D^N=A$

Example:

The logarithmic equation:

$Log_5 125=3$

The Exponential equation:

$5^3=125$

Write in logarithmic form:

1) $8^2=64$

2) $3^4=81$

3) $2^2=4$

4) $4^{-2}=\frac{1}{16}$

5) $12^{-1}=\frac{1}{12}$

6) $9^{\frac{3}{2}}=27$

7) $16^{\frac{1}{2}}=4$

8) $8^{-\frac{2}{3}}=\frac{1}{4}$

9) $y=a^x$

10) $y=3^x$

11) $5^x=6$

12) $y=\left(\sqrt{5}\right)^x$

10-4

Write in Exponential form:

1) $\log_3 x = a$

2) $\log_a x = b$

3) $\log_b N = x$

4) $\log_r s = t$

5) $\log_2 \frac{1}{4} = -2$

6) $\log_{10} 10 = -2$

7) $\log_x x^2 = 2$

8) $\log_x \pi = 8$

9) $\log_2 4 = 6$

10) $\log_{27} 9 = \frac{2}{3}$

11) $\log_{\frac{1}{8}} 2 = -\frac{1}{3}$

12) $\log_{\frac{1}{9}} 3 = -\frac{1}{2}$

10 - 5

Name____________________________ Date______________

Logs Day 2

$\text{Log}_D A = N$ **$D^N = A$**

Solve for X:

1) $\log_2 X = 4$

2) $\log_5 X = 2$

3) $\log_3 X = -2$

4) $\log_7 X = 0$

5) $\log_{81} X = \frac{1}{2}$

6) $\log_9 X = -\frac{1}{2}$

7) $\log_6 X = 0$

8) $\log_{10} X = 7$

9) $\log_{64} X = \frac{2}{3}$

Solve for X:

1) $\log_X 9 = \frac{1}{2}$

2) $\log_X 16 = 2$

3) $\log_X 64 = 3$

4) $\log_X 3 = \frac{1}{2}$

5) $\log_X \frac{1}{3} = -\frac{1}{3}$

6) $\log_X 8 = -3$

7) $\log_X 27 = 3$

8) $\log_X \frac{1}{2} = -1$

9) $\log_X \sqrt{5} = \frac{1}{4}$

Solve for X:

1) $\log_3 27 = X$

2) $\log_2 16 = X$

3) $\log_8 4 = X$

4) $\log_8 32 = X$

5) $\log_2 8 = X$

6) $\log_{27} \frac{1}{9} = X$

7) $\log_5 \frac{1}{25} = X$

8) $\log_{49} \frac{1}{7} = X$

9) $\log_{\sqrt{2}} 4 = X$

Name________________________________ Date____________

Logs Day 3 Class work/ Homework

$\text{Log}_D A = N$ **$D^N = A$**

Solve for X:

1) $\log_2 X = 3$
2) $\log_2 X = 5$
3) $\log_3 X = 4$

4) $\log_8 X = \frac{2}{3}$
5) $\log_9 X = \frac{1}{2}$
6) $\log_9 X = \frac{3}{2}$

Solve for X:

1) $\log_X 36 = 2$
2) $\log_X 125 = 3$
3) $\log_X 5 = \frac{1}{2}$

4) $\log_X 2 = \frac{1}{3}$
5) $\log_X \frac{1}{4} = -1$
6) $\log_X 16 = -2$

Solve for X:

1) $\log_2 8 = X$
2) $\log_8 2 = X$
3) $\log_{27} 3 = X$

4) $\log_2 \frac{1}{2} = X$
8) $\log_3 \frac{1}{3} = X$
9) $\log_{\sqrt{3}} \frac{1}{3} = X$

Log Relationships

Day 4

Product Rule: $\log_b AB = \log_b A + \log_b B$

ex. Find $\log_6 12 + \log_6 3$

Quotient Rule: $\log_b \frac{A}{B} = \log_b A - \log_b B$

ex. Expand: $\log_{10} \frac{a}{2b}$

Power Rule: $\log_b A^c = c\log_b A$

ex. Expand: $\log_{10} \frac{x}{\sqrt{y}}$

Apr 8-7:28 AM

1. $\log_9 4 + \log_9 6$
2. $\log_{12} 12 + \log_{12} 11$
3. $\log_{16} 36 - \log_{16} 12$
4. $\log 3 - \log 2$
5. $\log 14^6$
6. $\log_{20} 10^{16}$
7. $\log_3 16 + \log_2 4$
8. $\log 10 + \log 10$
9. $\log 125$
10. $\log_2 2^4$

Apr 8-8:38 AM

condense and solve

1. $\log_3 27 + \log_3 81$
2. $\log_3 6561 - \log_3 243$

condense: (write as one log)

3. $\log_e x + \log_e 10$
4. $\log_e x + 2\log_e y - 2\log_e z$

expand: (write as many logs as there variables and numbers)

5. $\log_4 \frac{x^6}{y^5}$

Apr 8-8:42 AM

Name________________________________ Date________________

Logs day 4

Laws of logarithms

1) Product rule: $\log_b AC = \log_b A + \log_b C$

2) Quotient Rule: $\log_b \frac{A}{C} = \log_b A - \log_b C$

3) Power Rule: $\log_b A^C = C \log_b A$

1) Find the value of:

$\log_6 12 + \log_6 3$

2) Solve for b:

$\log_4 b - \log_4 \frac{1}{4} = \log_4 64$

3) Solve for n:

$\log_2 10 + \log_2 4 = \log_2 n$

4) Solve for a:

$2\log_3 9 = \log_3 a$

Practice: Solve for n:

1) $\log_3 9 + \log_3 3 = \log_3 n$

2) $\log_4 64 - \log_4 16 = \log_4 n$

3) $3\log_2 4 = \log_2 n$

4) $\log_6 216 - \frac{1}{2}\log_6 36 = \log_6 n$

5) $\log 1000 - 2\log 100 = \log n$

6) $\log_3 n - \log_3 \frac{1}{3} = \log_3 9$

Name____ Date__________________

Logs day 4

Homework

Laws of logarithms

1) Product rule: $\log_b AC = \log_b A + \log_b C$

2) Quotient Rule: $\log_b \frac{A}{C} = \log_b A - \log_b C$

3) Power Rule: $\log_b A^C = C \log_b A$

1) Find the value of:
$\log_b 3 + \log_b X = \log_b 12$

2) Solve for X:
$\log_2 X + \log_2 4 = \log_2 32$

3) Solve for X:
$\log_4 5 + \log_4 (X - 2) = \log_4 70$

4) Solve for X:
$2\log_5 X - \log_5 5 = \log_5 125$

1) $\log_3 9 + \log_3 3 = \log_3 n$

4) $2\log X - \log(X-1) = \log 4$

2) $\log X + \log(X-3) = \log 10$

5) $3\log 2 - \log X = \log 16$

3) $\log_2 X + \log_2 (X+1) = \log_2 12$

6) $\log_2 (X-3) + \log_2 (X+1) = \log_2 32$

Name______________________________ Date___________

Logs day 5

Expanding and Contracting Logs

Contract the equation and solve for X in terms of a, b, c

1) $\log X = \frac{1}{2}(\log a + \log b - \log c)$	2) $\log X = \frac{1}{2}(\log a - (\log b + \log c))$
3) $\log X = 2\log a - \frac{1}{2}(\log b + \log c)$	4) $\log X = \frac{1}{2}\log a - (\log b + \frac{1}{2}\log c)$
5) $\log X = 2\log a + \frac{1}{3}\log b$	6) $\log X = 2\log a - \log b$
7) $\log X = \log a - \frac{1}{2}\log b$	8) $\log X = \frac{1}{2}(\log a + \log b)$

Expand and express log N in terms of log x, log y, log z :

1) $N = xyz$	2) $N = \sqrt{xyz}$
3) $N = \frac{xy}{z}$	4) $N = x^2 y \sqrt{z}$
5) $N = \frac{x^2 y}{z^3}$	6) $N = \sqrt{\frac{xy}{z}}$
7) $N = \frac{x\sqrt{y}}{z^2}$	8) $N = \frac{x^2}{y\sqrt{z}}$
9) $N = x^2(\sqrt[3]{y})$	10) $N = \sqrt[3]{x^2 y}$

DAY 6

Using Logarithms to solve $B^x = A$

'old' equation $4^x = 8$

'new' equation $3^x = 5$

RULES:
- a. log both sides
- b. use power rule
- c. get x by itself to solve

Apr 9-9:41 AM

Solve and round to the nearest tenth:

1) $6^x = 239$

2) $3^{2x} = 108$

3) $4.83^x = 29.1$

Jul 7-11:12 AM

Name________________________________ Day 6 Date____________

Class work

1) **Solve for X to the nearest tenth:**

a) $5^x=7$ **b)** $8^x= 29$ **c)** $7^x= 512$

d) $11.2^x=8.8$ **e)** $12^x = 23.2$ **f)** $5.8^x = 10.7$

2) **Using logarithms, find X to the nearest tenth.**

a) $5^{2x}=12$ **b)** $2^{3x}=7$ c) $1.73^{2x} = 9$

d) $6^{3x-1} = 74$ e) $5^{x+1} = 20$ f) $8^{2x+2} = 1000$

3) Solve for x to the nearest tenth:

a) $5^x - 2=7$ b) $3^{2x} - 2 = 8$ c) $5^x - 18 = 34$

4) Use logarithms to fins x to the nearest tenth:

a) $X= \log_5 29$ **b)** $X= \log_2 9$ **c)** $X= \log_2 32$

Name: ______________________________

Day 6

Class work/Homework

1) Solve for x to the nearest tenth: $3^x = 16$

2) Solve for x to the nearest tenth: $4^x = 28$

3) Solve for x to the nearest tenth: $6^{2x-1} = 73$

4) Solve for x to the nearest tenth: $2^{x-1} = 15$

5) Solve for x to the nearest tenth: $12^x = 215$

6) Solve for x to the nearest tenth: $4^x = 32.8$

7) Solve for x to the nearest tenth: $5^x - 18 = 34$

8) Solve for x to the nearest tenth: $1.62^{2x} = 8$

9) Solve for x to the nearest tenth: $20^x = 53$

10) Solve for x to the nearest tenth: $(41)^x = 3{,}000$

11) Solve for x to the nearest tenth: $4^{x+1} = 23$

12) Solve for x to the nearest tenth: $1.3^x + .8 = 5.3$

13) Using logarithms, solve the equation $2^{3x} = 7$ for x to the nearest tenth.

14) Which logarithmic equation is equivalent to $L^m = E$?

A) $\log_E m = L$

B) $\log_E L = m$

C) $\log_L E = m$

D) $\log_m E = L$

15) The expression log 12 is equivalent to

A) $\log 3 \cdot \log 4$

B) $\log 3 - 2 \log 2$

C) $\log 3 + 2 \log 2$

D) $\log 6 + \log 6$

16) The expression $\log 4x$ is equivalent to

A) $(\log 4)(\log x)$

B) $4 \log x$

C) $\log x^4$

D) $\log 4 + \log x$

17) The expression $\log \frac{x^2y^3}{\sqrt{z}}$ is equivalent to

A) $2 \log x + 3 \log y - \frac{1}{2} \log z$

B) $\log 2x + \log 3y - \log \frac{1}{2}z$

C) $\frac{(2x)(3y)}{\frac{1}{2}z}$

D) $2 \log x + 3 \log y + \frac{1}{2} \log z$

10-18

18) The expression $\frac{1}{3}\log(a) - 3\log(b)$ is equivalent to

A) $\log(\sqrt[3]{a} - b^3)$

B) $\log \frac{a}{3b^3}$

C) $\log \frac{\sqrt[3]{a}}{3b}$

D) $\log \frac{\sqrt[3]{a}}{b^3}$

19) The expression $2\log_5 m + \log_5 n$ is equivalent to

A) $\log_5 m^2 n$

B) $\log_5 (\frac{2m}{n})$

C) $\log_5 (\frac{m^2}{n})$

D) $\log_5 \sqrt{mn}$

20) Complete the following sentence:

To take the log of a product,

A) take the difference of the log of the numerator and the denominator
B) take the sum of the logs of the two factors
C) square the product of the two logs
D) take the product of the logs of the two factors

21) Complete the following sentence:

To take the log of a quotient,

A) take the quotient of the logs of the two factors
B) take the quotient of the log of the numerator divided by the log of the denominator
C) take the sum of the logs of the numerator and the denominator
D) take the difference of the log of the numerator and the log of the denominator

22) Express $\log x$ in terms of $\log a$, $\log b$, and $\log c$:
$x = a \cdot b$

23) Express $\log x$ in terms of $\log a$, $\log b$, and $\log c$:
$x = \frac{a}{bc}$

24) Express $\log x$ in terms of $\log a$, $\log b$, and $\log c$:
$x = a^2 b$

25) Express $\log x$ in terms of $\log a$, $\log b$, and $\log c$:
$x = \frac{(ab)^3}{c}$

26) Express $\log x$ in terms of $\log a$, $\log b$, and $\log c$:
$x = \frac{\sqrt{ab}}{c^2}$

27) If $\log x = 2\log a + 2\log b - \frac{1}{2}\log c$, then express x in terms of a, b, and c.

28) If $\log x = \frac{1}{3}\log a + 2\log b + \log c$, then express x in terms of a, b, and c.

10-19

DAY 7

Word Problems

In the following examples use the formula $A = P(1 + r/n)^{nt}$

13. How long must $500 be left in an account that pays 7% interest compounded annually in order for the value of the account to be $750?

Apr 12-1:28 PM

$A = P(1 + r/n)^{nt}$

14. How long must a sum of money be left in an account at 6% interest compounded semiannually (twice a year) in order to double?

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$A = P(1 + r/n)^{nt}$

15. How long must $100 be left at 8% interest compounded quarterly (four times a year) in order to acquire the value $1,000?

Apr 12-1:35 PM

16. The thickness of a sheet of paper is .004 inch. If x represents the number of times that this sheet of paper is folder in half over itself, then $y = 2^x$ determines the number of layers of paper, and $y = .004(2)^x$ determines the thickness of all the layers of paper. Calculate the number of folds that would produce a stack of paper closest to a mile high.
(1 mile = 63, 360 inches)

Apr 12-1:36 PM

17. When Patty was in kindergarten , her mother gave her 10 cents a week to spend. In the first grade, Patty received 20 cents a week, double her kindergarten allowance. In the third grade, Patty suggested to her mother that her allowance be doubled every year, but her mother was wise enough to refuse. If Patty's suggestion had been followed, in what grade would her weekly allowance have been more than $200?
(Hint: Use the formula $y = .10(2^x)$)

Apr 12-1:40 PM

Name: ______________________________

Day 7/8

1) A radioactive material decays according to the formula $A = A_0 10^{-kt}$ where A is the final amount, A_0 is the initial amount, and t is time in years. Find k, if 500 grams of this material decays to 450 grams in 10 years. [*Round the answer to 4 decimal places.*]

A) 0.0046
B) 1.1065
C) -16.9897
D) -0.9000

2) A radioactive material decays according to the formula $A = A_0 10^{-kt}$ where A is the final amount, A_0 is the initial amount, and t is time in years. Find k, if 700 grams of this material decays to 550 grams in 8 years. [*Round the answer to 4 decimal places.*]

A) -4.4820
B) 0.1179
C) 0.0131
D) -0.1179

3) The growth of a colony of cells can be determined by the formula $G = I(3.1)^{0.226t}$, in which G represents the final number in the colony, I is the initial number of cells, and t represents elapsed time in hours. Find how many hours it will take for a colony starting at 25 cells to increase to a size of 25,000 cells. [*Round the answer to the nearest whole hour.*]

4) The growth of a certain strain of bacteria is given by the equation $C = I(2.4)^{0.621t}$, where C is the final number of bacteria, I is the initial number of bacteria, and t is the number of hours. If the initial number of bacteria was 7, find the numbers of hours required for the colony to reach 3200 bacteria. [*Round the answer to the nearest tenth of an hour.*]

5) The value V of a savings account in which interest is compounded annually can be determined by the formula $V = C(1 + r)^t$, where C represents the amount of the initial deposit, r is the rate of interest, and t is the number of years for which the balance has been accruing interest. If $1,500 was deposited in 2001 at an annual interest rate of 5%, what is the first year that the account will be worth $3,000? [*Assume that only interest is added to the account.*]

6) It has been shown that homes in a certain city increase in value at a rate of 7.5% per year. The value V of a home after t years is given by the formula $V = C(1 + r)^t$ where r is the rate of appreciation. If a home costs $42,000 in 2001, by what year will this home have doubled in value?

7) A new boat will decrease in value at a rate of 8% per year according to this formula $V = C(1 - r)^t$ where V is the value of the boat after t years, C is the original cost, and r is the rate of depreciation. If a boat costs $40,000 new, find the number of years until the boat is worth $18,000. [*Round the answer to the nearest tenth of a year.*]

8) During surgery, a patient must have *at least* 40 mg of an antibiotic in his system. The amount of antibiotic present k hours after administration of 100 mg of this antibiotic is given by $P(k) = 100(.508)^k$. After how many hours (to the nearest tenth) will the nurse have to administer another dose of the antibiotic to keep the level of antibiotic high enough?

9) A basketball is dropped from a height of 9 ft. Each time it bounces, it returns to a height of 65% of its previous height. The height h may be determined by the formula $h = 9(.65)^n$ where n is the number of bounces. Find the number of bounces it will take for the ball to reach a height of *no more* than 1.5 ft.

10) A super bouncy ball is dropped from a height of 12 ft. Each time it bounces, it rises to a height of 80% of the height from which it fell. The height h can be determined by the equation $h = 12(.80)^x$, where x is the number of bounces. Determine the number of bounces necessary for the ball to be *at most* 2 ft from the floor.

11) The Richter Scale measures the magnitude R of an earthquake. It is defined by the formula $R = 0.67 \log(0.37E) + 1.46$ where E is the energy (in kilowatt-hours) released by the quake. The 1933 quake in Japan measured 8.9 on the scale. In scientific notation (with 3 significant digits), how much energy was released?

12) The Richter Scale measures the magnitude R of an earthquake. It is defined by the formula $R = 0.67 \log(0.37E) + 1.46$ where E is the energy (in kilowatt-hours) released by the quake. The 1960 quake in Morocco measured 5.8 on the scale. In scientific notation (with 3 significant digits), how much energy was released?

13) An exponential model of a population growth is given by $P(t) = P_0 \cdot 10^{kt}$ where P_0 equals the original or initial population and t equals the number of years that have elapsed. If a population of a culture is 2,000 now and is 4,500 in 2 years then what is the value of k?

14) Suppose the exponential model of a population growth is given by $P(t) = P_0 \cdot 10^{kt}$ where P_0 equals the original or initial population and t equals the number of years that have elapsed. If a population of a culture is 1,500 now and is 2,400 in 2 years then what is the value of k?

15) The amount of money A after t years that principle P will become if it is invested at rate r compounded n times a year is given by the relationship $A(t) = P(1 + \frac{r}{n})^{nt}$ where r is expressed as a decimal. To the nearest tenth, how long will it take \$2,500 to become \$4,500 if it is invested at 7% and is compounded quarterly?

16) The amount of money A after t years that principal P will become if it is invested at rate r compounded n times a year is given by the relationship $A(t) = P(1 + \frac{r}{n})^{nt}$ where r is expressed as a decimal. To the nearest tenth, how long will it take \$5,300 to become \$7,000 if it is invested at 9% and is compounded quarterly?

17) The amount of money A after t years that principal P will become if it is invested at rate r compounded n times a year is given by the relationship $A(t) = P(1 + \frac{r}{n})^{nt}$ where r is expressed as a decimal. To the nearest tenth, how long will it take \$3,600 to become \$5,200 if it is invested at 9% and is compounded semi-annually?

18) The amount of money A after t years that principal P will become if it is invested at rate r compounded n times a year is given by the relationship $A(t) = P(1 + \frac{r}{n})^{nt}$ where r is expressed as a decimal. To the nearest tenth, how long will it take \$2,700 to become \$4,200 if it is invested at 7% and is compounded semi-annually?

10-25

19) The amount of money A after t years that principal P will become if it is invested at rate r compounded n times a year is given by the relationship $A(t) = P(1 + \frac{r}{n})^{nt}$ where r is expressed as a decimal. To the nearest tenth, how long will it take a sum of money to double if it is invested at 12% and compounded annually?

20) The amount of money A after t years that principal P will become if it is invested at rate r compounded n times a year is given by the relationship $A(t) = P(1 + \frac{r}{n})^{nt}$ where r is expressed as a decimal. To the nearest tenth, how long will it take a sum of money to double if it is invested at 9% and compounded annually?

Name: ____ Period: _______

Day 9 Test Review

1) Find the value of $\log_8 4$.

2) Solve for x to the nearest tenth: $1.3^x + .8 = 5.3$

3) Write the equation in exponential form:
$\log_2 \frac{1}{4} = -2$

4) Solve for x to the nearest tenth: $4^x = 28$

5) Simplify: $(-3np)(4n^2p^2)$

6) If $\log_9 x = \frac{3}{2}$, what is the value of x?

A) 8
B) $\frac{27}{2}$
C) $\frac{3}{2}$
D) 27

7) If $\log_x 9 = \frac{1}{2}$ what is the value of x?

A) $4\frac{1}{2}$
B) 27
C) 81
D) 3

8) What is the exponential form for $\log_a x = b$?

A) $b = a^x$
B) $x = a^b$
C) $a = b^x$
D) $b = x^a$

9) The expression $\log 4x$ is equivalent to

A) $\log 4 + \log x$
B) $(\log 4)(\log x)$
C) $\log x^4$
D) $4 \log x$

10) $\text{Log} \frac{\sqrt{a}}{\sqrt{b}}$ is equivalent to

A) $\frac{1}{2} \log a + \log b$
B) $\frac{1}{2}(\log a + \log b)$
C) $\frac{1}{2}(\log a - \log b)$
D) $\frac{1}{2} \log a - \log b$

10-27

11) The expression $\log \frac{x^2y^3}{\sqrt{z}}$ is equivalent to

A) $\frac{(2x)(3y)}{\frac{1}{2}z}$

B) $\log 2x + \log 3y - \log \frac{1}{2}z$

C) $2 \log x + 3 \log y - \frac{1}{2} \log z$

D) $2 \log x + 3 \log y + \frac{1}{2} \log z$

12) Solve for x: $\log 64 = 2 \log x$

A) 5
B) 8
C) 4
D) 1

13) Simplify: $\frac{3^{x+4}}{3^x}$

A) 81
B) $\frac{1}{81}$
C) -81
D) $-\frac{1}{81}$

14) Write the equation in logarithmic form: $3^4 = 81$

15) Solve: $\log_4 (3x + 1) = 2$

16) It has been shown that homes in a certain city increase in value at a rate of 7.5% per year. The value V of a home after t years is given by the formula $V = C(1 + r)^t$ where r is the rate of appreciation. If a home costs $42,000 in 2001, by what year will this home have doubled in value?

17) An exponential model of a population growth is given by $P(t) = P_0 \cdot 10^{kt}$ where P_0 equals the original or initial population and t equals the number of years that have elapsed. If a population of a culture is 2,000 now and is 4,500 in 2 years then what is the value of k?

10-28